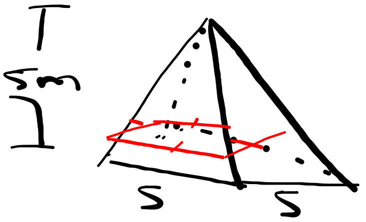
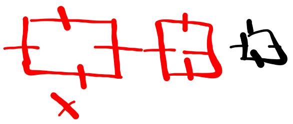
7.3 Volumes

The volume of a solid if known integrable cross section area from x=a to x=b is:

$$V=\int_a^b A(x)\,dx$$

Ex. 1 A pyramid 5m high has congruent triangular sides and a square base that is 5m on all sides. Each cross section of the pyramid parallel to the base is a square. Find the volume of the pyramid.





$V=\frac{1}{3}bh$

$=\frac{1}{3}5\cdot5\cdot5$

$=125/3\ m^3$

$\int_0^5 x^2\,dx$

$\left.\frac{x^3}{3}\right]_0^5$

$\frac{125}{3}\ m^3$

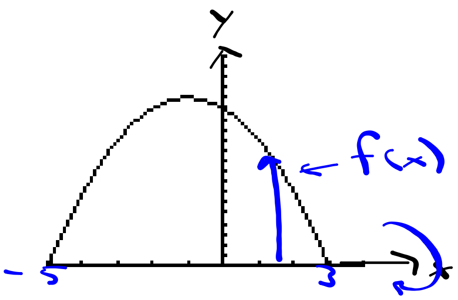
CALCULUS BC

The only thing that changes when the cross sections of the solid are circular is the area formula. These are called solids of revolution.

$$A(x) = \pi(f(x))^2$$

$$A = \pi r^2$$

Ex. 2 The region between the graph of $f(x) = 15 - 2x - x^2$ and the x-axis over the interval [-5,3] is revolved around the x-axis to generate a solid. Find the volume of the solid.

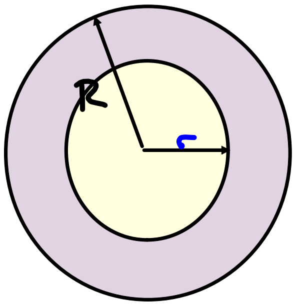


$$\pi \int_{-5}^{3} (15 - 2x - x^2)^2 \, dx$$

$$\underline{\frac{16384}{15} \pi}$$

CALCULUS BC

Washer Method

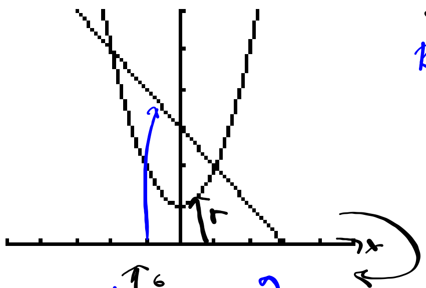


$A_B = \pi R^2$

$A_L = \pi r^2$

$A_s = \pi(R^2 - r^2)$

Ex. 3 The region enclosed by the graphs of $y=x^2+1$ and $y= -x+3$ is revolved about the x-axis to form a solid.
a) Find the volume.
b) Repeat, but revolve around the line y = 6.
c) Repeat, but revolve around the line y = -2.

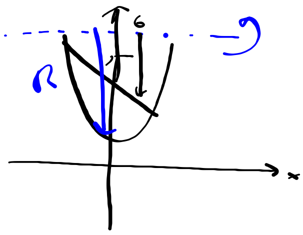


$r = x^2+1$

$R = -x+3$

$V = \pi \int_{-2}^{1} \left((-x+3)^2 - (x^2+1)^2\right) dx$

$\frac{117}{5}\pi$ units^3

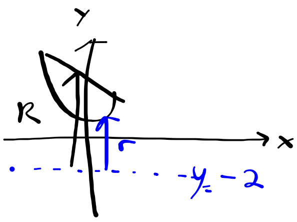


$r = 6-(-x+3)$

$R = 6-(x^2+1)$

$V = \pi \int_{-2}^{1} \left((6-(x^2+1))^2 - (6-(-x+3))^2 \right) dx$

$= \frac{153}{5}\pi$ units^3



$R = (-x+3) - (-2)$

$r = (x^2+1) - (-2)$

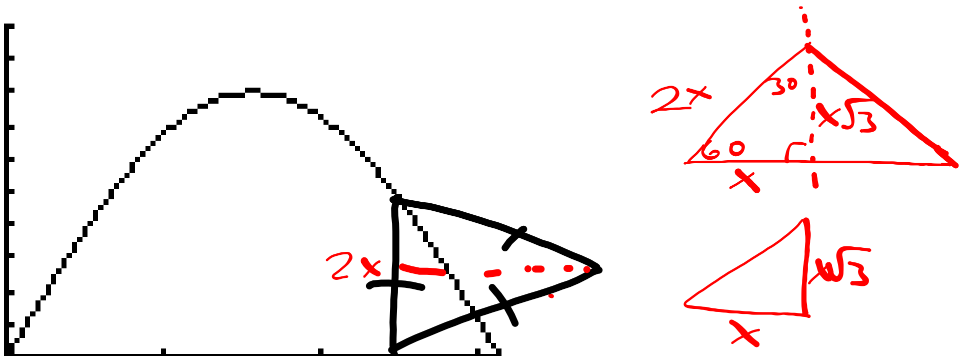
$V = \pi \int_{-2}^{1} \left[((-x+3)+2)^2 - (x^2+1+2)^2 \right] dx$

$= \frac{207}{5}\pi$

CALCULUS BC

Perpendicular Cross sections and Volume

Ex. 4 A base of a paperweight is the shape of the region between the x-axis and the curve y=8sinx. Each perpendicular cross-section is an equilateral triangle. Find the volume of the paper weight.



$2x = 8\sin x$

$'x' = 4\sin x$

$A_{\frac{1}{2}\Delta} = \frac{1}{2} x \, x\sqrt{3}$

$= \frac{1}{2} 16\sqrt{3} \sin^2 x$

$A_{1\Delta} = 2 \cdot \frac{1}{2} 16\sqrt{3} \sin^2 x$

$= 16\sqrt{3} \sin^2 x$

$$V = 16\sqrt{3} \int_0^{\pi} \sin^2 x \, dx$$

$$= \frac{16\sqrt{3}}{2} \int_0^{\pi} (1 - \cos 2x) \, dx$$

$$= 8\sqrt{3} \left[x - \frac{1}{2}\sin 2x \right]_0^{\pi}$$

$$= 8\sqrt{3}\pi$$

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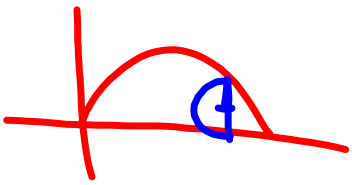
CALCULUS BC

TRY A base of a snow globe is the shape of the region between the x-axis and the curve $y = 4 - (x-2)^2$. Each perpendicular cross-section is a semicircle. Find the volume of the snowglobe.



$A_1 = \frac{1}{2}\pi r^2$

$\Rightarrow r = \left(\frac{1}{2}\left(4-(x-2)^2\right)\right)^2$

$V = \frac{\pi}{2}\int \left(\frac{1}{2}\left(4-(x-2)^2\right)\right)^2 dx$

$= \frac{64}{15}\pi \text{ units}^3$

An alternative to the disk and washer method is cylindrical shells. The volume for cylindrical shells is

V=∫2π(shell radius)(shell height)dx

or

V=∫2π(shell radius)(shell height)dy

It is the radius that tells us our integrating variable.

Strategy

Draw the cylinder parallel to the axis of revolution.

The radius will be perpendicular to the axis of revolution.

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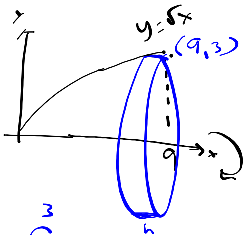
Ex. 5 The region bounded by the curves $y=\sqrt{x}$ and x=9 is rotated about the x-axis to form a solid. Using cylindrical shells find the volume of the solid. Compare your answer using the disk method. Repeat rotating about the line x=6.



$y=\sqrt{x}$

$y^2=x$

$r=y$

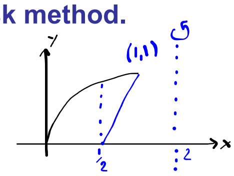
$h=9-x$

$=9-y^2$

$$2\pi\int_0^3 y(9-y^2)\,dy$$

$$\frac{81\pi}{2}$$

Ex 6 The region bounded by the curves y=2x-1 and $y=\sqrt{x}$ is rotated about the line x=2 to form a solid. Using cylindrical shells find the volume of the solid. Compare your answer using the disk method.



$r=2-x$

$h_1=\sqrt{x}$ $\quad h_2=\sqrt{x}-(2x-1)$

$$2\pi\int_0^{\frac{1}{2}}(2-x)\sqrt{x}\,dx+2\pi\int_{\frac{1}{2}}^{1}(2-x)(\sqrt{x}-(2x-1))\,dx$$

$\sqrt{2}\pi \approx 4.442$

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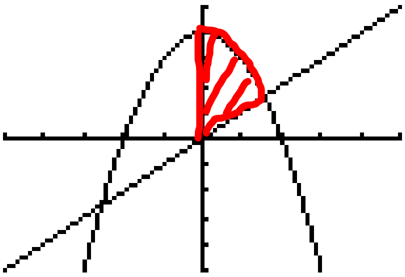
CALCULUS BC

TRY The region in the first quadrant bounded by the curves $y=4-x^2$, $y=x$ is rotated about the y-axis to form a solid. Using cylindrical shells find the volume of the solid.



TRY Find the volume of the solid generated by the curve $y=3x-x^2$ and the positive x axis, when the region is revolved about the line x=-1.

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